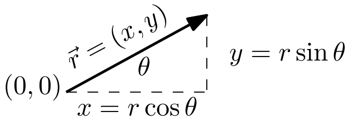
OnRamps Physics: Mechanics, Heat, and Sound



Radians versus degrees. $\theta_{\text{radians}} = \theta_{\text{degrees}}\pi/180$

Arc Length: $s = r\Delta\theta$, $r = \sqrt{r_x^2 + r_y^2}$

$\theta = \text{atan}(r_y/r_x) = \tan^{-1}(r_y/r_x)$

Quantity	*Units*
Position	m
Time	s
Mass	kg
Velocity	m/s
Acceleration	m/s^2
Energy	Joules = kg m^2/s^2
Force	Newtons = kg m/s^2 = Joule/m

$$\text{Velocity: } \bar{v} = \frac{\Delta x}{\Delta t} \quad v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$$\text{Acceleration: } a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

One-dimensional accelerated motion

$$\begin{aligned} \text{Uniform acceleration: } x &= x_0 + v_0 t + \frac{1}{2}at^2 \\ \text{Velocity: } v &= v_0 + at \\ \text{Velocity and distance: } v^2 &= v_0^2 + 2a(x - x_0) \end{aligned}$$

Two-dimensional uniformly accelerated motion. In the absence of air resistance, for uniformly accelerated motion in two dimensions, the x and y directions can be treated independently.

$$\begin{aligned} \text{Free fall: } y &= y_0 + v_0 t - \frac{1}{2}gt^2 \\ v_x &= v_{0x} \quad \Delta x = v_{0x}t \\ \Delta x &= v_0 \cos\theta_0\, t \quad v_y = v_{0y} - gt \\ \Delta y &= v_{0y}\, t - \frac{1}{2}gt^2 \\ \Delta y &= v_0 \sin\theta_0\, t - \frac{1}{2}gt^2 \\ v_y^2 &= v_0^2 \sin^2\theta_0 - 2g\Delta y \\ \text{Range: } \quad \Delta x &= \frac{v_0^2 \sin 2\theta_0}{g} \end{aligned}$$

$$\text{Newton's second law: } \quad \vec{F} = m\vec{a}$$

$$\text{Newton's third law: } \quad \vec{F}_{12} + \vec{F}_{21} = 0$$

Force due to static friction is less than or equal to $\mu_s N$. Force due to kinetic friction is $\mu_k N$.

Mechanical energy is conserved for systems without external forces, and which do not generate heat through friction, and do not create or consume chemical energy.

$$\begin{aligned} \text{Kinetic Energy: KE} &= \frac{1}{2}mv^2. \\ W_{\text{net}} &= \Delta\text{KE} \\ U_{\text{gravity}} &= mgh; \quad U_{\text{spring}} = \frac{1}{2}kx^2 \end{aligned}$$

Power is defined as work per time

$$\text{Momentum: } \vec{p} = m\vec{v}$$

Momentum is conserved for any system for which total external force vanishes.

$$\begin{aligned} \Delta p &= I = F\Delta t \\ m_1 v_{1\text{before}} + m_2 v_{2\text{before}} &= m_1 v_{1\text{after}} + m_2 v_{2\text{after}} \\ \text{Elastic}: v_{1\text{before}} - v_{2\text{before}} &= v_{2\text{after}} - v_{1\text{after}} \\ \text{Inelastic}: v_{2\text{after}} &= v_{1\text{after}} \\ \text{Rocket Thrust}: & \quad (\Delta m/\Delta t) v_{\text{ejectedgas}} \\ \text{Rocket Speed}: v &= v_{\text{ejectedgas}} \ln(m_0/m_f) \end{aligned}$$

Rotational Motion.

$$\begin{aligned} \Delta\theta &= \theta_f - \theta_i \\ \text{Angular velocity: } \omega &= \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} \\ \text{Tangential velocity:} v &= r\omega \\ \text{Angular acceleration: } \alpha &= \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} \\ \text{Tangential acceleration:} a &= r\alpha \\ \text{Const. angular accel.: } \omega &= \omega_0 + \alpha t \\ \Delta\theta &= \omega_0 t + \frac{1}{2}\alpha t^2 \\ \omega^2 &= \omega_0^2 + 2\alpha\Delta\theta \end{aligned}$$

$$\begin{aligned} \text{Centripetal accel.: } a_c &= \frac{v^2}{r} \\ \text{Gravitation: } F &= G\frac{m_1 m_2}{r^2} \\ \text{Torque: } \tau &= rF\sin\theta \\ \text{Center of Gravity: } x_{\text{cg}} &= \frac{\sum_i m_i x_i}{\sum_i m_i} \\ \text{Moment of Inertia: } I &= \sum_i m_i r_i^2 \end{aligned}$$

Hoop	$I = MR^2$
Solid Sphere	$I = \frac{2}{5}MR^2$
Thin Spherical Shell	$I = \frac{2}{3}MR^2$
Solid Cylinder	$I = \frac{1}{2}MR^2$
Thin Rod (Center)	$I = \frac{1}{12}MR^2$
Thin Rod (End)	$I = \frac{1}{3}MR^2$

Angular momentum is conserved for any system where the total torque vanishes. Results can depend on choice of origin.

$$\begin{aligned}\text{Torque: } \tau &= I\alpha \\ \text{Angular Kinetic Energy: } \mathrm{KE}_{\mathrm{rot}} &= \frac{1}{2}I\omega^2 \\ \text{Angular Momentum: } L &= I\omega\end{aligned}$$

Stress is Force/Area, F/A. Strain is $\Delta L/L_0$

$$\begin{aligned}\text{Young's Modulus:} \frac{F}{A} &= Y\frac{\Delta L}{L} \\ \text{Density:} \rho &= M/V; \quad \text{Pressure:} P = F/A \\ P_2 &= P_1 + \rho g(y_1 - y_2)\end{aligned}$$

Water has density of 1000 kg/m^3. 1 liter (L) is 1 m^3/1000 = 1000cm^3. Atmospheric pressure is 1.013×10^5 Pa. Archimedes Principle: The upward force on any stationary object partially or completely submerged in non-moving water is equal to the weight of the displaced water.

$$\begin{aligned}\text{Continuity:} &\quad v_1A_1 = v_2A_2; \\ \text{Bernoulli's Law:} &\quad P + \frac{1}{2}\rho v^2 + \rho gh = \text{const}\end{aligned}$$

Zeroth Law of Thermodynamics: Objects in equilibrium are at same temperature

$$\begin{aligned}T_F &= \frac{9}{5}T_C + 32; T = T_C + 273.16 \\ \text{Thermal Expansion:}\Delta L &= \alpha L_0 \Delta T \\ N_A &= 6.02\times10^{23}\text{particles/mole} \\ \text{Ideal Gas:} PV &= Nk_BT; PV = nRT \text{ (Kelvin)}\end{aligned}$$

$$P = \frac{2}{3}\left(\frac{N}{V}\right)\frac{1}{2}m\overline{v^2} \qquad \frac{1}{2}m\overline{v^2} = \frac{3}{2}k_BT \quad U = \frac{3}{2}nRT$$

$$\text{Specific Heat: } cm\Delta T = Q \quad \text{Latent heat } mL = Q$$

Second Law of Thermodynamics: Heat always flows from hot to cold

Thermal Conductivity:

$$Q = \frac{\kappa A(T_h - T_c)}{L} = \frac{A(T_h - T_c)}{\sum_i R_i}, \; R = \frac{L}{\kappa}$$

First Law of Thermodynamics: Energy is conserved. W is $-PV$(area)

$$\Delta U = U_f - U_i = Q + W$$

$$\begin{aligned}\text{Efficiency; } e &= \frac{|W|}{|Q_h|} = \frac{|Q_h| - |Q_c|}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|} \\ e_C &= 1 - \frac{T_c}{T_h} \\ \text{COP} &= \frac{|Q_c|}{|W|} = \frac{|Q_c|}{|Q_h| - |Q_c|} \leq \frac{T_c}{T_h - T_c}.\end{aligned}$$

Process	ΔU	Q	W	P-V
General	$nC_V\Delta T$	$\Delta U - W$	PV Area	$P(V)$
Isobaric	$nC_V\Delta T$	$nC_P\Delta T$	$-P\Delta V$	P =const
Adiabatic	$nC_V\Delta T$	0	ΔU	PV^γ =const$^{(1)}$
Isovolumetric	$nC_V\Delta T$	ΔU	0	V =const
Isothermal	0	$-W$	$-nRT\ln\left(\frac{V_f}{V_i}\right)$	PV =const

$^{(1)}\gamma = C_P/C_V$

$$\text{Simple Harmonic Motion: } F(x) = ma = -kx$$

$x(t)$ and $v(t)$ are the same as if the mass is traveling around a circle of radius A with velocity $v_{\max} = 2\pi A/T = \omega A$, but one keeps track only of motion in the x direction.

$$\begin{aligned}x &= A\sin(2\pi t/T) \quad v = v_{\max}\cos(2\pi t/T) \\ \omega &= 2\pi f = 2\pi/T = \sqrt{k/m} \\ \text{Pendulum: } F &= -mg/L \Rightarrow \omega = \sqrt{g/L} \\ T &= 2\pi\sqrt{L/g}\end{aligned}$$

$$\begin{aligned}\text{Waves: } v &= \frac{\lambda}{T} = f\lambda \\ \text{Transverse wave on string: } v &= \sqrt{F/\mu}\end{aligned}$$

- Superposition:Two waves traveling through each other simply add point by point.
- Waves reflecting off fixed ends reverse direction and amplitude.
- Waves reflecting off free ends reverse direction but not amplitude.

Longitudinal waves:

$$\begin{aligned}\text{Bulk modulus: } B &= -\frac{\Delta P}{\Delta V/V} \\ v &= \sqrt{B/\rho} \quad v = \sqrt{Y/\rho} \\ \text{For sound in air: } v &\approx 331 \text{ m/s}\sqrt{\frac{T}{273K}}\end{aligned}$$

For a string fixed at both ends.

$$f_1 = v/\lambda_1 = v/(2L) = \frac{1}{2L}\sqrt{F/\mu}$$

$$f_n = nf_1 = \frac{n}{2L}\sqrt{F/\mu}, \text{ where } n = 1, 2, 3\ldots$$

Here n describes the n'th harmonic.

For a standing wave in an air column closed at one end and open at the other,

$$f_n = nf_1 = \frac{nv}{4L}, \text{ where } n = 1, 3, 5\ldots$$