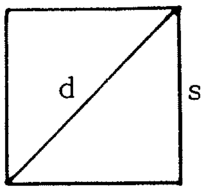
APPENDIX C: Geometric Formulas

(Refer to the underlined letters for the following as well.)

Square

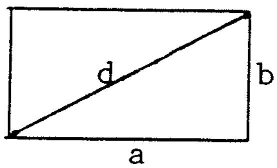


Perimeter = $4s = 4(d/\sqrt{2})$ side = $d/\sqrt{2} = P/4 = \sqrt{A}$

Area = $s^2 = d^2/2 = (P/4)^2$

diagonal = $s\sqrt{2} = \sqrt{A/2}$

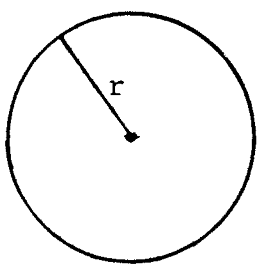
Rectangle



Perimeter = $2a + 2b = 2(a + b)$ diagonal = $\sqrt{a^2 + b^2}$

Area = $(a)(b)$ side a = $\sqrt{c^2 - b^2}$

Circle



Diameter = $2r$

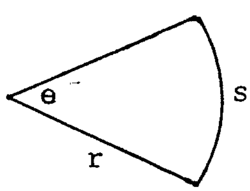
Circumference = $2\pi r = \pi D = \sqrt{4\pi A}$

Area = $\pi r^2 = \pi D^2/4 = C^2/(4\pi)$

radius = $D/2 = \sqrt{A/\pi} = C/(2\pi)$

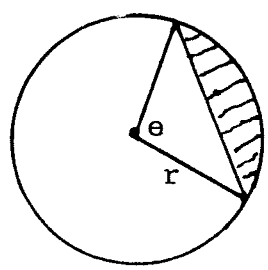
Circular Sector

[θ in radians]



$s = r\theta$ Area = $\frac{1}{2}r^2\theta$

Circular Segment

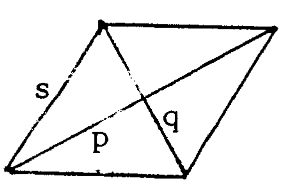


Area = $\frac{1}{2}r^2(\theta - \sin\theta)$

[θ in radians]

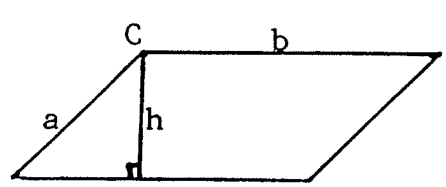
radius = $\sqrt{2A/(\theta - \sin\theta)}$

Rhombus



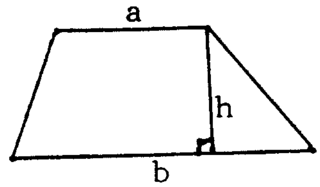
Area = $\frac{1}{2}pq$ Perimeter = $4s$

Parallelogram



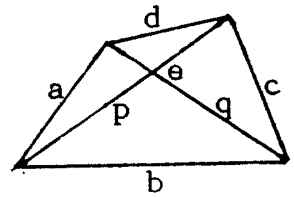
Area = $bh = ab\sin C$ Perimeter = $2(a + b)$

Trapezoid



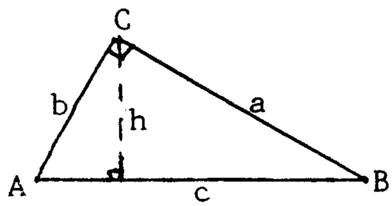
Area $= \frac{1}{2}(a+b)h$

General Quadrilateral



Area $= \frac{1}{2}pq\sin\theta = \frac{1}{4}(b^2+d^2-a^2-c^2)\tan\theta$

Right Triangle



$\angle A + \angle B = \angle C = 90° = \pi/2$ rad. $\quad c^2 = a^2 + b^2$

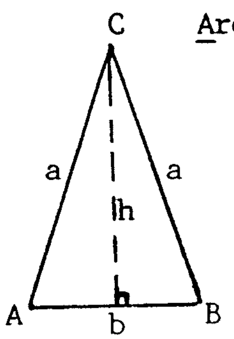
$c = \sqrt{a^2+b^2} \quad a = \sqrt{c^2-b^2} \quad b = \sqrt{c^2-a^2}$

Area $= \frac{1}{2}ab = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B \quad h = ab/c$

$\sin A = a/c \quad \cos A = b/c \quad \tan A = a/b$

$\sin B = b/c \quad \cos B = a/c \quad \tan B = b/a$

Isosceles Triangle



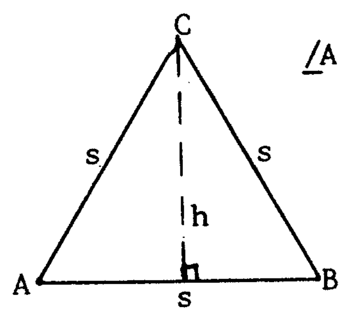
Area $= \frac{1}{2}bh$

$= \frac{1}{2}a^2\sin C$

$= \frac{1}{2}ab\sin A$

$\angle A = \angle B$

Equilateral Triangle



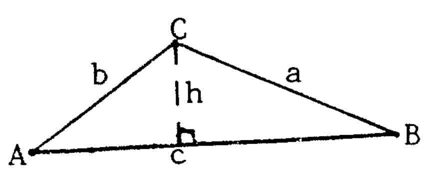
$\angle A = \angle B = \angle C = 60°$

$h = \frac{s\sqrt{3}}{2}$

Area $= \frac{s^2\sqrt{3}}{4}$

$= \frac{h^2\sqrt{3}}{3}$

Scalene Triangle



$\angle A + \angle B + \angle C = 180° = \pi$ rad.

Area $= \frac{1}{2}hc = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B =$

$= \sqrt{s(s-a)(s-b)(s-c)}$, $s = \frac{1}{2}(a+b+c)$

$c = \sqrt{a^2+b^2-2ab\cos C} \quad a = \sqrt{c^2+b^2-2cb\cos A} \quad b = \sqrt{a^2+c^2-2ac\cos B}$

Trigonometric Functions and Identities

$\sin A = \dfrac{\text{side opposite } \angle A}{\text{hypotenuse}}$

$\cos A = \dfrac{\text{side adjacent } \angle A}{\text{hypotenuse}}$

$\tan A = \dfrac{\text{side opposite } \angle A}{\text{side adjacent } \angle A}$

Law Of Sines: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

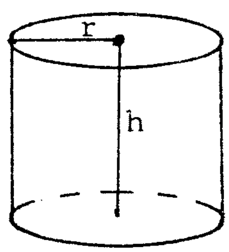
Law Of Cosines: $c^2 = a^2 + b^2 - 2ab\cos C$

degree measure = (radian measure)$(180°/\pi)$

radian measure = (degree measure)$(\pi/180°)$

*** For the following, LA = lateral area, T = total surface area, and V = volume.

Right Circular Cylinder

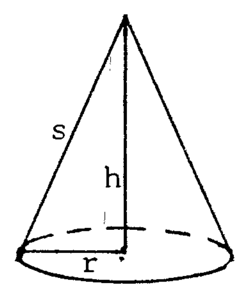


$LA = 2\pi rh$

$T = 2\pi r(r + h)$

$V = \pi r^2 h$

Right Circular Cone



$s = \sqrt{r^2 + h^2}$

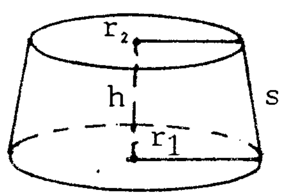
$LA = \pi rs$

$T = \pi r(r + s)$

$V = \frac{1}{3}\pi r^2 h$

Frustrum Of Cone

[B_2 = upper base area]
[B_1 = lower base area]



$s = \sqrt{(r_1 - r_2)^2 + h^2}$

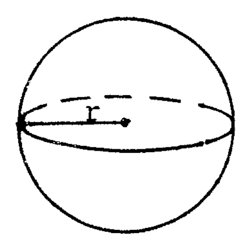
$LA = \pi s(r_1 + r_2)$

$T = \pi[r_1^2 + r_2^2 + (r_1 + r_2)s]$

$V = \frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1 r_2]$

$= \frac{1}{3}h(B_1 + B_2 + \sqrt{B_1 B_2})$

Sphere

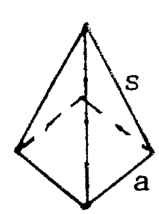


Diameter = 2r

Surface Area $= 4\pi r^2 = \pi D^2$

Volume $= \frac{4}{3}\pi r^3 = \frac{1}{6}\pi D^3$

Pyramid



$LA = 2as$

$T = 2as + a^2$

$V = \frac{1}{3}a^2 h$

h(altitude) $= \sqrt{s^2 - a^2/2}$

Section Overview

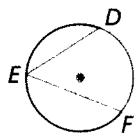
Professional Development

Inscribed Angles

Lessons 11-4

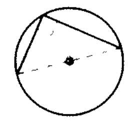
Why? Understanding inscribed angles allows you to solve application problems.

$\angle DEF$ is an inscribed angle.
$\widehat{DF}$ is an intercepted arc.
$\widehat{DF}$ *subtends* $\angle DEF$.

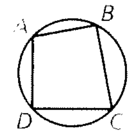


Inscribed Angle Theorem
The measure of an angle inscribed in a circle is half the measure of its intercepted arc. $m\angle DEF = \frac{1}{2} m\widehat{DF}$

An inscribed angle subtends a semicircle if and only if the angle is a right angle.



A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.
$\angle A$ and $\angle C$ are supplementary.
$\angle B$ and $\angle D$ are supplementary.

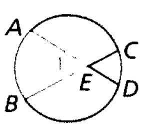


Angle and Segment Relationships in Circles

Lessons 11-5, 11-6

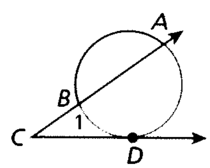
Why? Many professional fields of study, such as ophthalmology and archaeology, use angle and segment relationships in circles.

Segments intersecting inside the circle

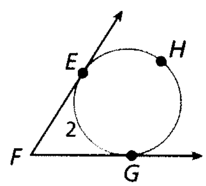


$m\angle 1 = \frac{1}{2}\left(m\widehat{AB} + m\widehat{CD}\right)$

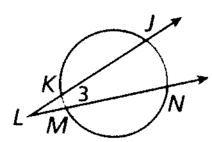
Segments intersecting outside the circle



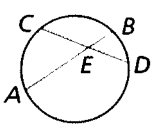
$m\angle 1 = \frac{1}{2}\left(m\widehat{AD} - m\widehat{BD}\right)$



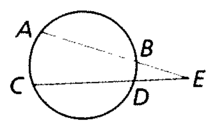
$m\angle 2 = \frac{1}{2}\left(m\widehat{EHG} - m\widehat{EG}\right)$



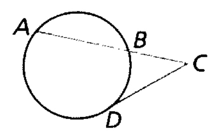
$m\angle 3 = \frac{1}{2}\left(m\widehat{JN} - m\widehat{KM}\right)$



If chords $\overline{AB}$ and $\overline{CD}$ intersect at E, then $AE \cdot EB = CE \cdot ED$.



If secants $\overline{AE}$ and $\overline{CE}$ intersect at E, then $AE \cdot BE = CE \cdot DE$.



If secant $\overline{AC}$ and tangent $\overline{DC}$ intersect at C, then $AC \cdot BC = DC^2$.

Circles in the Coordinate Plane

Lessons 11-7

Why? The location of weather stations, radio towers, and radar devices can be planned by using circles in the coordinate plane.

Equation of a Circle

$$(x - h)^2 + (y - k)^2 = r^2$$

r = radius
(h, k) = **coordinates of the center**